Abstract—This study presents an effective approach based on Schur decomposition in frequency domain for removing speckle noise from ultrasound images. The proposed scheme has been tested on both simulated and real ultrasound images, and is compared with different benchmark schemes including the Schur Regular, PNLM and Lee. The theme of proposed approach is to segment a speckle noise corrupted image into various overlapping blocks of a small size, and generate the global covariance matrix by averaging the covariances of individual blocks. The Fourier transform of global covariance matrix is taken and Schur decomposition is applied to generate eigenvectors. These frequency domain orthogonal vectors are arranged in a descending order, and a subset is used to create the Feature matrix, which is used for speckle noise removal. The proposed approach shows better performance than benchmark techniques in terms of both Peak Signal to Noise Ratio (PSNR) and Signal to Noise Ratio (SNR), which are regarded as key parameters in the despeckling area.

Keywords—feature matrix, global covariance matrix, speckle noise, Ultrasound image

I. INTRODUCTION

Medical imaging plays vital role to diagnose and monitor the diseases. Ultrasound is a real time, non-invasive and radiation free medical imaging modality unlike MRI, CT scan and X-rays [1]. During image acquisition process, the ultrasound images inherit multiplicative noise which extensively degrades the image quality by reducing contrast, corrupting edges and other details, important for accurate diagnosis [2].

Extensive research has been done since last decade to address this critical issue. Proposed despeckling methods can be categorized in to spatial domain and frequency domain based methods. Spatial traditional despeckling methods include Kuan [3], Lee [4], Frost [5], diffusion based filters [6], etc. These traditional methods carry the disadvantage of losing edge based information of the image while carrying higher number of speckles on edges. Wavelets are the most famous frequency based method in which multiplicative noise is converted into additive noise which is removed within wavelet domain. Many variants of this technique have been proposed in [7]–[9]. The main disadvantage of wavelet is the artifacts generation within image due to mother wavelet which degrades image quality.

In [10] quantum based despeckling method is proposed in which multiplicative noise is detected by quantum probability theory then thresholding is used to separate it. Another quantum based approach is proposed in [11] where adaptive bilateral filter based on quantum signal processing is used in pre-processing stage for better speckle detection. Non Local Mean (NLM) based techniques proved very efficient as well in removing multiplicative noise. In this method a window is considered instead of individual pixel and every single pixel is recovered on the basis of weighted average of the whole window [12] and [13]. Later multiple modifications were proposed to address the drawbacks of this technique e.g. traditional NLM did not consider the variability factor of speckle noise as compared to Gaussian noise, in [14] refine weights are used to address this issue. Probabilistic Non Local Mean (PNLM) used Bayesian instead of postulations of relevant pixels within patches [15]. In [16] NLM is used with optimized Bayesian where distance of similarity between two patches is calculated by Pearson distance unlike Euclidean distance, which is used in traditional NLM. A hybrid of NLM and local statistical model is proposed in [17] where statistical characteristics of the speckle noise from pre-processing stage are used for denoising. Although better performance is achieved through NLM based methods but these all techniques mainly rely on grey level information of the image and features to calculate the distance of similarity between patches and in case of rough speckle noise such methods may not work.

Recently, projection based methods are being used to remove the speckle noise from ultrasound images. In such methods overlapped segments are used to exploit the eigenvector characteristics of the covariance matrix [18]. The performance of these methods depends on the calculation and selection of eigenvectors from global covariance matrix of the image, in [19] Schur based calculation of eigenvectors is exploited for better results and it showed improved performance as compared to latest benchmarks, yet the selection of eigenvector is user dependent. In this research, we will address this issue by incorporating frequency domain analysis with Schur based eigenvectors. The Section II explains about developing an ultrasound signal model and quality measurement of ultrasound images, Section III presents steps of proposed algorithm for suppressing speckle noise, and Section IV includes the results for both simulated and real ultrasound images. Section V concludes the paper.

II. ULTRASOUND SIGNAL MODEL AND ASSESSMENT OF IMAGE QUALITY PARAMETERS

A generalized noise model usually includes both additive and multiplicative noise but multiplicative noise (speckle) is comparatively more difficult to remove. A noisy image based model has been presented in [20], which is commonly used in ultrasound images.

\[ I(x,y) = a(x,y)b(x,y) + c(x,y) \]  \hspace{1cm} (1)

where, \( I \) denotes envelope of the observed image, \( a \) is the original image, \( b \) denotes speckle noise and \( c \) denotes Additive White Gaussian Noise (AWGN). In ultrasound imagery, the AWGN part can be ignored. So, (1) can be reduced to,
\[ I(x,y) = a(x,y)b(x,y) \] (2)

This paper utilizes a simplified model only containing the speckle noise as presented in (2). Let us assume that \( I(x,y) \) is observed prior to applying any pre-processing such as non-linear amplification and log-compression as explained in [21] and [22].

There are many pseerformance measuring parameters used in assessing despeckling capability of the algorithms but most commonly used are \( \alpha, \beta, SNR, PSNR, S-SNR \) and \( S-\text{SNR} \) [19], [23]–[25]. Parameter \( \alpha \) denotes resolution of the image and lower value corresponds to better resolution. \( \beta \) denotes edge detection/sharpness of image and its value close to 1 corresponds to better performance in terms of preserving edges. \( \text{CNR} \) denotes Contrast-to-noise ratio and higher value indicates clearer identification of different tissue parts. \( \text{SNR}, \text{PSNR} \) and \( S-\text{SNR} \) denote Signal to Noise ratio, Peak Signal to Noise ratio and Speckle Signal to Noise ratio respectively.

III. PROPOSED ALGORITHM BASED ON SCHUR DECOMPOSITION IN FREQUENCY DOMAIN

The following algorithm summarizes steps of suppressing speckle noise through sub-space signal projection based on Schur decomposition in the frequency domain.

Step 1: Given an ultrasound image with speckle noise having envelope size \( xy \), segment it into overlapping segments (s) each of size \( pxq \), where \( i \) denotes the index of respective segments.

Step 2: Reshape segments (s) having size \( pxq \) into size \( p.qx1 \) and evaluate the segment’s contribution to covariance matrix \( v_i \), such that \( v_i = sxsT \), where \( T \) denotes transpose.

Step 3: Generate a Global Covariance Matrix \( R \) of size \( p.qxp.q \) by adding all individual \( v_i \) matrices and afterwards averaging them by the total number of segments.

Step 4: Apply Fourier transformation on the overall covariance \( R \) to obtain the Fourier Transform (FT) matrix.

Step 5: Apply Schur decomposition on \( FT \) to obtain the eigenvalues and eigenvectors of \( FT \) matrix.

Step 6: Sort the eigenvectors based on the magnitudes of their relevant eigenvalues in a descending manner, such that the first column of features vector \( F \) contains eigenvector corresponding to the highest eigenvalue. Create a features matrix \( F \) by choosing the first \( k \) orthogonal vectors.

Step 7: Compute Projection matrix \( P \) such that \( P = FXF^T \). Then apply Fourier transformation on the segment \( s \) from Step 1 and reshape it to \( p.qx1 \).

Step 8: Project \( (p.qx1) \) segment from \( P \) in order to get denoised segment \( v \) so that \( v = Pxs \), and size of \( v \) is \( p.qx1 \).

Step 9: Perform reshaping of denoised segment \( v \) back to \( pxq \) size, and apply inverse Fourier transform. Finally, image is being rebuilt by summing all updates of overlapping segments and averaging each sample by number of updates.

IV. RESULTS

A. Simulated Ultrasound Image

Field II program is used in order to create a simulated noise-free ultrasound image of abdomen region [26], as presented in Fig. 1-(a). The speckle noise removal ability of proposed algorithm is assessed by applying linear scaling on noise-free image having size \( 256 \times 256 \). Afterwards image is corrupted with speckle noise by (2), as depicted in Fig 1-(b). The despeckling algorithms are applied to noise corrupted image Fig. 1-(b), and outcomes of proposed and benchmark schemes are compared with the noise-free image Fig. 1-(a).

The parameters are being set to produce best visual results for both proposed and benchmark schemes. The proposed Schur decomposition in frequency domain algorithm has been implemented in an overlapping design with each block having size of \( 8 \times 8 \). The proposed technique is compared with benchmark algorithms including Schur Regular [19], PNLM [15] and Lee [4] filters. The despeckled images are depicted in Fig. 1-(c), (d), (e) and (f) after applying the speckle noise removal algorithms i.e. Proposed, Schur Regular, PNLM and Lee respectively. The numerical outcomes are enlisted in Table I after averaging outcomes of 50 independent trials. Proposed approach outperformed all benchmarking algorithms with reference to \( SNR, PSNR \) and \( CBR \) while maintaining reasonable \( \alpha, \beta \) and \( S-SNR \).

Any despeckling algorithm can be regarded as efficient, if it maintains \( \alpha \) and \( S-SNR \) at a low value, whereas maintains the \( SNR, PSNR \) and \( CBR \) high. From the visual and numerical results given in Fig. 1 and Table I respectively, the benchmark algorithms used in this paper i.e. Schur Regular, PNLM and Lee showed better performance in case of \( \alpha \) but at the cost of lower despeckling performance and edge detection. Our proposed algorithm showed tremendous improvement with respect to \( SNR \) and \( PSNR \), as shown in Table I which is the main criteria to assess despeckling efficiency and it is clear from the visual results depicted in Fig. 1-(c) as well.

![Fig. 1. Ultrasound image simulated case: (a) Noise Free, (b) Speckled, (c) Schur Frequency, (d) Schur Regular, (e) PNLM, (f) Lee](image-url)

<table>
<thead>
<tr>
<th></th>
<th>Schur Freq.</th>
<th>Schur Regular</th>
<th>PNLM</th>
<th>Lee</th>
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<tr>
<td>( \alpha )</td>
<td>0.0834</td>
<td>0.0810</td>
<td>0.0390</td>
<td>0.0799</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.2639</td>
<td>0.2305</td>
<td>0.1718</td>
<td>0.1804</td>
</tr>
<tr>
<td>SNR</td>
<td>18.7185</td>
<td>15.9756</td>
<td>16.1730</td>
<td>15.4843</td>
</tr>
<tr>
<td>PSNR</td>
<td>26.3660</td>
<td>24.4960</td>
<td>23.7575</td>
<td>23.8975</td>
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<tr>
<td>S-SNR</td>
<td>1.5847</td>
<td>1.4655</td>
<td>1.7646</td>
<td>1.4758</td>
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</table>

The other impressive thing about proposed method is its performance in terms of edge preservation. Usually better despeckling is secured at cost of decrease in sharpness but
the proposed method achieved better results in terms of $\beta$, as shown in Table I. The benchmark schemes PNLM and Schur Regular have shown better performance in terms of $\alpha$ and S-SNR respectively but at the cost of poor despeckling as per visual and numerical outcomes of Fig. 1 and Table I respectively. Overall performance of the proposed algorithm shall be regarded as best because it supersedes other benchmark algorithms in cleaning the speckle noise i.e. in terms of PSNR and SNR with reasonable resolution $\alpha$.

Fig. 2 presents the comparative analysis of lateral and axial profiles which were taken from the middle of image. The proposed technique shows best approximation to the profile of noise-free image in both lateral and axial directions. The benchmark schemes such as Schur Regular and Lee tend to suffer significant variation in comparison to noise free image profiles. Although PNLM based profiles are similar to lateral and axial profiles of noise-free image, but contain abrupt spikes throughout. Hence, proposed technique has shown better performance in profile analysis as well.

B. Real Ultrasound Image

In this paper, the real images of tumor and gallbladder have been used after cropping to the size of 256x256, as available at [27]. Fig. 3-(a) and Fig. 4-(a) represent the real images of tumor and gallbladder respectively. Similar despeckling parameters which were used in simulated ultrasound image for proposed and other benchmarking schemes are used for real ultrasound images because better despeckling results were achieved with such parameters for simulated images as well. The numerical outcomes are enlisted in Table II and Table III, pertaining to ultrasound images of tumor and gallbladder respectively. For real ultrasound images, the parameters such as SNR, PSNR and $\beta$ cannot be used, because of unavailability of noise free image. Therefore, only $\alpha$, CNR and S-SNR have been used to compare the performance of proposed and benchmark speckle denoising schemes.
The visual results of Fig. 3 and Fig. 4 depict that proposed scheme has efficiently removed speckle noise from real ultrasound images while maintaining reasonable α and S-SNR as per numerical results of Table II and Table III. Best CNR is achieved for both images as per Table I and Table II which is also reflected in the visual results of Fig. 3-(b) and Fig. 4-(b). Although PNLM has shown better numerical results in terms of CNR for tumor image as shown in Table II, but the analysis of visual results of PNLM in Fig. 3-(d) clearly depicts the presence of speckle noise. Similar kind of results are observed with gallbladder images where proposed technique cleaned speckle noise with reasonable resolution. With Schur Regular and PNLM, the best resolution is achieved as per numerical analysis in Table II and Table III, but image got over-smoothed as depicted in Fig. 3 and Fig. 4 respectively.

Also, similar trends in numerical results for simulated images were observed in Table I i.e. better α and S-SNR does not guarantee better despeckling. Thus, by comparing both the visual outcomes (Fig. 3 and Fig. 4) and the numerical outcomes (Table II and Table III), it can be deduced that the proposed scheme is more efficient in removing speckle noise from real ultrasound images in comparison to the benchmark schemes.

The proposed algorithm’s performance depends on total count of the orthogonal vectors and block size selected in order to create the projection matrix. This study yields that the best performance is achieved through block size of 8x8 and selecting the first four orthogonal vectors. Other variations such as less number of orthogonal vectors and large block size produce an over-smoothed image, whereas large number of orthogonal vectors and small block size tend to cause inefficient despeckling. In order to minimize the complexity of the proposed algorithm the despeckling of ultrasound image shall be done separately in both the lateral and axial directions.

V. CONCLUSION

A speckle noise removal scheme based on schur decomposition in frequency domain has been proposed. Based on the outcomes of simulated and real ultrasound images, it has been observed that Schur decomposition of the global covariance matrix in the frequency domain has a better despeckling performance when compared to the performances of benchmark schemes i.e. Schur Regular, PNLM and Lee. Visual results show a high degree of speckle noise removal while using the proposed approach. Numerical results also maintain superior values in terms of SNR, PSNR, and maintain the image resolution and edge detection at reasonable levels. The lateral and axial profile data about middle points also justifies that proposed approach has better speckle removal ability and a closer resemblance to the noise-free image.

TABLE II

<table>
<thead>
<tr>
<th>Schur Freq.</th>
<th>Schur Regular</th>
<th>PLMN</th>
<th>Lee</th>
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<tbody>
<tr>
<td>α</td>
<td>0.0822</td>
<td>0.0875</td>
<td>0.0621</td>
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<tr>
<td>CNR</td>
<td>0.2826</td>
<td>0.2281</td>
<td>0.3012</td>
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<td>S-SNR</td>
<td>1.9257</td>
<td>1.8648</td>
<td>1.8525</td>
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TABLE III

<table>
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<th>Schur Freq.</th>
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<th>PLMN</th>
<th>Lee</th>
</tr>
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<tbody>
<tr>
<td>α</td>
<td>0.0792</td>
<td>0.0748</td>
<td>0.0533</td>
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<tr>
<td>CNR</td>
<td>0.3388</td>
<td>0.3308</td>
<td>0.3264</td>
</tr>
<tr>
<td>S-SNR</td>
<td>1.5966</td>
<td>1.5137</td>
<td>1.5494</td>
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REFERENCES


